THE FRICTION COEFFICIENT IN NONSTATIONARY FLOWS

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The results are given of an experimental investigation into the coefficient of hydraulic friction drag for an accelerated turbulent flow in a pipe.

In various areas of modern technology ever more frequently we meet problems requiring the calculation of sharply nonstationary processes in machines and apparatuses and in particular the calculation of nonstationary flows of a fluid in pipes. But whereas the laws of stationary flows have been studied in a more or less satisfactory manner, the same cannot be said for nonstationary flows although they have long attracted the attention of students of hydromechanics.

Back in 1915 Bakhmet'ev concluded from a consideration of the physical pattern of nonstationary flow that pressure head losses (due to friction) in accelerated motion must be greater, and for a retarded flow, less, than for a stationary flow [1].

However, subsequent papers for a long time were unable to refine this question; on the contrary, their conclusions were contradictory.

In the last 10-15 years the authors of the experimental investigations [2] and [3] stated that the friction coefficients in nonstationary flows differ somewhat from those in stationary flows.

In [4, 5] this distinction was very marked both for turbulent and for laminar flows, analytic expressions being obtained for the friction coefficients to laminar flows.

An experimental expression was proposed in [6] for turbulent flows:

$$\lambda = \lambda_0 - 1.28 \frac{D}{w^2} \frac{dw}{dt}, \tag{1}$$

where λ_0 is the friction coefficient for an "established flow."*

But the coefficient of nonstationary friction and even the reduced coefficient of nonstationary friction in general cannot be determined by the criterion

$$\Lambda = \frac{\lambda}{\lambda_q}. \tag{2}$$

Here λ_q is the quasistationary value of the friction coefficient, computed or measured for stationary friction conditions at the same Reynolds number as the nonstationary flow has at the given moment of time. Obviously $\lambda_q = f(t)$ is a function of one variable only for sufficiently large times and only for a particular class of functions defining the change in the velocity (for example, for linear and power functions) [4].

Since the concept of a nonstationary friction coefficient is still not generally accepted, and frequently is linked with dissipative losses, we consider briefly the general formulation of the problem.

Most practical problems in the hydromechanics of nonstationary flows involve the establishment of the relation between the flow rate G(t) and the pressure drop $\Delta P(t)$. These problems, and also problems in the expenditure of power "on pumping" are solved by considering the equations of motion which express the fact that the change in fluid momentum is equal to the sum of the impulses of the forces acting on it.

*Which established flow in particular was taken to calculate λ_0 in [6] was not made clear.

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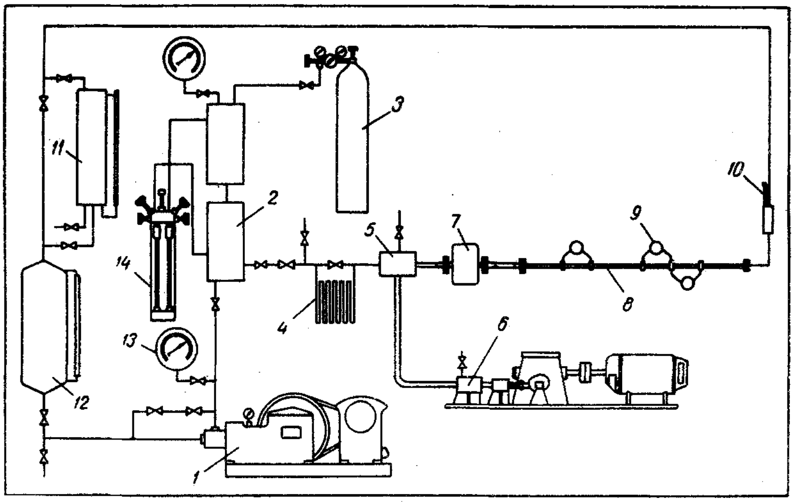

Fig. 1. Diagram of the experimental apparatus: 1) pump; 2) reservoir; 3) air cylinder; 4) capillary throttle; 5) mixing chamber; 6) flow rate control; 7) flow meter; 8) working section; 9) pressure drop detector; 10) thermometer; 11) measuring tank; 12) feed tank; 13) manometer; 14) differential manometer.

The equation of motion of an incompressible fluid in a cylindrical pipe with rigid walls, written in terms of the projection on the axis

$$\int_{x_1}^{x_2} \frac{\partial}{\partial t} \int_F u dF dx + \int_F \{[u(x_2)]^2 - [u(x_1)]^2\} dF = \frac{1}{\rho} \int_F [P(x_1) - P(x_2)] dF - \frac{1}{\rho} \int_{x_1}^{x_2} \int_{\chi} \tau_w d\chi dx, \tag{3}$$

can be reduced to the following form after introducing variables averaged over a section and round the perimeter:

$$\Delta P = (\Delta x) \rho \frac{dw}{dt} + \frac{4}{D_h} \int_{x_1}^{x_2} \tau_w dx + [\beta(x_2) - \beta(x_1)] \rho w^2. \tag{4}$$

The second term on the right side of (4), expressing the pressure losses due to friction can be put in the form

$$\Delta P_{fr} = \lambda \frac{|\rho w| w}{2} \frac{\Delta x}{D}, \tag{5}$$

where λ is the coefficient of nonstationary friction.

For flows in a pipe of circular cross-section

$$\lambda = \frac{8\tau_w}{\rho w^2}. \tag{6}$$

We note that λ does not determine the energy dissipation. This is clear from the example of flows in which the velocity changes from one direction to the opposite. For such flows $\lambda = 0$ at some point in time and for certain time intervals $\lambda < 0$, although there is always dissipation, which is, of course, always positive.

To determine λ experimentally in segments where the flow is stabilized along its length (where $\beta(x_2) = \beta(x_1)$), it is not necessary to measure the tangential friction stress at the wall or the velocity profile. From (4) and (6) we have

$$\lambda = \frac{\left(\frac{\Delta P}{\Delta x} - \rho \frac{dw}{dt}\right) 2D}{\rho w^2}, \tag{7}$$

i.e., it is sufficient to measure $w(t)$ (or $G(t)$) and $\Delta P(t)$ in the experiments.

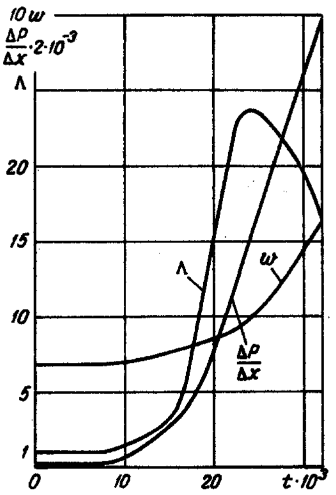

Fig. 2

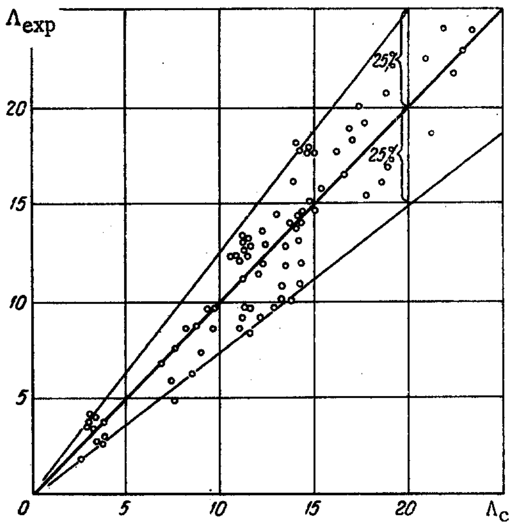

Fig. 3

Fig. 2. Graph of the change with time of the velocity w, m/sec, the pressure gradient $\Delta P/\Delta x$, bar/m, and the reduced friction coefficient Λ.

Fig. 3. Comparison of experimental and computed (from (9)) values of Λ.

It is expedient [4, 5] to seek an equation not directly for λ, but for its ratio to the quasistationary value λ_q (see (2), i.e., for the reduced nonstationary friction coefficient Λ.

In the experimental investigation described below the reduced nonstationary friction coefficient was studied for accelerated turbulent water flows in a pipe in a section where they were stabilized along the length of the pipe.

The experimental apparatus was a closed circuit (see Fig. 1). Water from a pump (type Tl-200) at a pressure of 120-150 bar was fed into a reservoir (to eliminate fluctuations), from which it was admitted to the working section through a throttle and a mixing chamber at a pressure of P = 2-5 bar, the working section being a seamless pipe of 1Kh18N9T steel of thickness 1.5 mm, internal diameter 10 mm and length 2.44 m.

This ensured a constant flow rate through the working section. A perturbation (variable flow rate) was pumped by a special flow rate control in the form of a cylinder with a piston set in motion in a particular manner by a cam mechanism. The piston injected an additional quantity of water into the mixing chamber and the total flow passed through the working section. Although the pressure in the mixing chamber varied with the time (depending on the flow rate of the injected water), the constant component of the flow rate was virtually unchanged as a result of the large drop at the throttle (capillary and needle valves).

The flow rate of the water through the working section was measured (at the inlet to the section) by an induction flow meter based on the flow meter ZRI-25 with an electrical current carrier frequency of 500 Hz.*

The pressure drop ΔP was measured in a segment of the pipe of length $\Delta x = 120$ mm at 1 m from the inlet. A pressure drop detector of type DIF-IM was used to measure ΔP. The impulse lines (of the pipe $\phi\, 8 \times 1$, $l = 40$ mm), as calculations showed, introduced an error not exceeding 1% of the measured pressure drop.

Before each series of experiments the induction flow meter and the pressure drop detector DIF-IM were calibrated using ID-2I amplifiers and an MPO-2 oscillograph in steady state conditions.

In the experiments to determine Λ the acceleration of the flow was varied from 8 to 140 m/sec^2 and the initial velocity from 0.7 to 1.3 m/sec, while the mean velocity at the end of the accelerated regime varied up to 2.7 m/sec. The water temperature in the experiments was 25-30°C.

*The flow meter was modernized by engineers L. A. Santalov and V. Ya. Gurovich.

The oscillograms of the nonstationary processes were first processed and the data on the MPO-2 oscillograph tape calibrated using a P10 enlarger.

The experimental results were later processed on a digital computer.

Since the experimental values of the velocity and the pressure drop (in a time interval of 0.002 sec) had random scatter, they were smoothed using Chebyshev approximation polynomials and the method of least squares [7]. From the smoothed curves for $w(t)$ and $\Delta P(t)$, values of dw/dt, d^2w/dt^2, Re, λ, Λ, K_1 and certain other variables were calculated.

The experimental values of the friction coefficient in the working section for stationary flows are well described (with a mean-square error of less than 1%) by Blasius' equation. Hence Λ was calculated from the equation

$$\Lambda = \frac{2D}{0.316}\,\frac{\mathrm{Re}^{0.25}}{\rho w^2}\left(\frac{\Delta P}{\Delta x} - \rho\frac{dw}{dt}\right). \tag{8}$$

Typical curves for the changes in $w(t)$, $\Delta P/\Delta x$, and Λ during the experiment are given in Fig. 2.

Analysis and processing of the experimental results showed that λ depends on criteria involving not only the first, but also the second derivative of the velocity with respect to the time.

As a result of processing the experimental results, an equation was obtained for the approximate calculation of Λ for the accelerated flow of water in pipes:

$$\Lambda = \exp(-20K_1) + 20K_1\,\frac{1+K_2}{1+10K_1}\exp(1+K_1), \tag{9}$$

where

$$K_1 = \frac{D}{w^2}\,\frac{dw}{dt}, \tag{10}$$

$$K_2 = \frac{1}{w}\sqrt[3]{D^2\frac{d^2w}{dt^2}}. \tag{11}$$

Since the above equation is empirical, without additional verification it cannot be extrapolated beyond the range of the parameters of the experiment. The parameters K_1 and K_2 in the experiment varied within the ranges

$$0.04 \leqslant K_1 \leqslant 0.43,$$

$$0.12 \leqslant K_2 \leqslant 0.86,$$

and Λ reached the value of 20.

The error in determining Λ from (9) in the above range of parameters is estimated at ±25% (see Fig. 3).

NOTATION

D	is the internal pipe diameter;
D_h	is the hydraulic diameter of pipe;
F	is the cross-sectional area of pipe;
G	is the mass flow rate;
P, ΔP	are the pressure (mean at section) and pressure drop in segment;
t	is the time;
u	is the local velocity;
w	is the mean flow rate;
$w = w(t)$;	
x	is the distance;
β	is the coefficient of velocity averaged with respect to momentum;
λ	is the coefficient of friction drag;
Λ	is the reduced coefficient of nonstationary friction drag;
ρ	is the density;

τ_w is the tangential stress at pipe wall;
χ is the pipe perimeter;
$Re = wD/\nu$ is the Reynolds number.

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